

2.7 Prove Angle Pair Relationships

Theorem 2.3--All right angles are congruent

Theorem 2.4--The Congruent Supplements Theorem--

If 2 \angle s are supplementary to the same \angle (or $\cong \angle$ s) then the \angle s are \cong

Theorem 2.5--The Congruent Complements Theorem--

If 2 \angle s are complementary to the same angle (or $\cong \angle$ s) then they are \cong

Prove Theorem 2.4

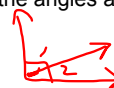
Given: $\angle 1$ and $\angle 2$ are supplementary $\angle 3$ and $\angle 2$ are supplementaryProve: $\angle 1 \cong \angle 3$

- | | |
|--|------------------|
| ① $\angle 1 + \angle 2$ are suppl.
$\angle 3 + \angle 2$ are suppl. | ① Given |
| ② $m\angle 1 + m\angle 2 = 180$
$m\angle 3 + m\angle 2 = 180$ | ② def of suppl. |
| ③ $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$ | ③ subst. |
| ④ $m\angle 2 = m\angle 2$ | ④ Refl. |
| ⑤ $m\angle 1 = m\angle 3$ | ⑤ subst. |
| ⑥ $\angle 1 \cong \angle 3$ | ⑥ def of \cong |

Postulate 12—The Linear Pair Postulate (L.P.P.)
If 2 \angle s form a linear pair then they are suppl.

The Complement Theorem—If the noncommon sides of 2 adjacent angles form a right angle, then the angles are complementary angles. (not in book)

Given: picture

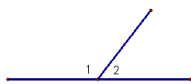
Conc: $\angle 1 + \angle 2$ are compl.

Theorem 2.6—Vertical angles are congruent



How they are used:

Given: picture



- | | |
|---|--------------------------------|
| 1. $\angle 1$ and $\angle 2$ are a linear pair. | 1. Definition of a linear pair |
| 2. $\angle 1$ and $\angle 2$ are supplementary | 2. The Linear Pair Postulate |
| 3. $m\angle 1 + m\angle 2 = 180$ | 3. Def. of supplementary |

Given: $\angle 1 \cong \angle 3$ Prove: $\angle 2 \cong \angle 4$

Statements

Reasons

1. $\angle 1 \cong \angle 3$

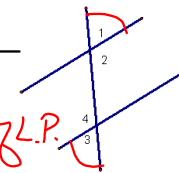
1. Given

2. $\angle 1 + \angle 2$ are a L.P.
 $\angle 3 + \angle 4$ are a L.P.

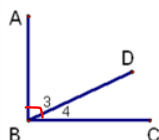
2. def of L.P.

3. $\angle 1 + \angle 2$ are suppl
 $\angle 3 + \angle 4$ are suppl

3. L.P.P.

4. $\angle 2 \cong \angle 4$ 4. \cong Suppl. ThmGiven: $\overline{AB} \perp \overline{BC}$

- | | |
|--|-------------------------|
| 1. $\angle ABC$ is a right angle | 1. Def of \perp lines |
| 2. $\angle 3$ and $\angle 4$ are complementary | 2. The Complement Thm. |

Given: $\angle 1$ and $\angle 2$ are supplementaryConclusion: $m\angle 1 + m\angle 2 = 180$

Reason: Definition of supplementary angles

Given: $\angle 1$ and $\angle 2$ are complementaryConclusion: $m\angle 1 + m\angle 2 = 90$

Reason: Definition of complementary angles

Given: $\angle 1$ is a right angleConclusion: $m\angle 1 = 90$

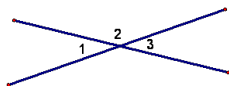
Reason: Definition of right angles

Given: $\overline{AB} \perp \overline{BC}$ Conclusion: $\angle ABC$ is a right angle

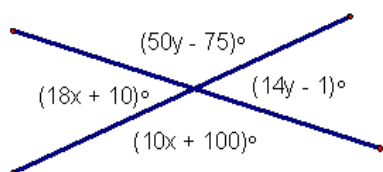
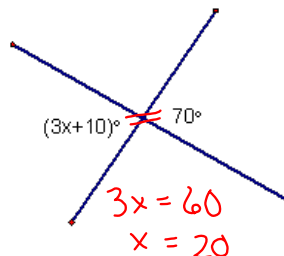
Reason: Definition of perpendicular lines

Proof of theorem 2.6:

Given: picture

Prove: $\angle 1 \cong \angle 3$ 

Statements	Reasons
① $\angle 1 + \angle 2$ are L.P. $\angle 2 + \angle 3$ are L.P.	① Def of L.P.
② $\angle 1$ & $\angle 2$ are suppl. $\angle 2$ & $\angle 3$ are suppl	② L.P.P.
③ $\angle 1 \cong \angle 3$	③ \cong Suppl. thm



$$50y - 75 + 14y - 1 = 180$$

$$64y = 256$$

$$y = 4$$

$$28x + 110 = 180$$

$$28x = 70$$

$$x = 2.5$$